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Engineering Note

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Payload Mass Fraction Optimization for Solar Sail Cargo Missions

Colin R. McInnes,* Gareth Hughes,†
and Malcolm Macdonald†
University of Glasgow,
Glasgow, Scotland G12 8QQ, United Kingdom

Introduction

SOLAR sailing offers the potential to reduce the required initial mass in low Earth orbit for future piloted Mars missions.[1–3] Athough solar sailing does not appear to be suitable for crew transport, it can be an extremely efficient mode of propulsion for the transport of logistics in support of a human crew. This may include premission caching of logistics and/or resupply missions to support long-duration surface stays. Because solar sails do not require reaction mass, a single solar sail may, in principle, be used for multiple Earth–Mars–Earth round trips. The limit to the number of round trips that can be made by a single solar sail will be dictated largely by the lifetime of the sail film in the space environment.

Previous studies of the solar sail cargo mission problem have considered either point designs[1] or have considered specific launch opportunities.[2] However, a key question that arises when considering the use of solar sails for round-trip logistic supply missions is the optimum payload mass fraction of the solar sail. As the payload mass fraction of the solar sail is increased, a greater payload mass is delivered, but the trip time will also increase. Similarly, as the payload mass fraction of the solar sail is decreased, a smaller payload is delivered, but with a shorter trip time. The payload mass fraction that is selected should, therefore, be chosen to balance these two effects and maximize the mean rate of payload mass transfer to Mars.

This Note provides a simple, quasi-analytic solution to the question of payload mass fraction optimization by using parametric trajectory data and a solar sail mass model. The total trip time from low Earth orbit (LEO) to low Mars orbit (LMO) is parameterized as a power law function of the sail performance. The power law is a fit to numerically generated minimum-time trajectories, although particular launch opportunities are not considered here. It is shown that an optimum payload mass fraction can be obtained that then maximizes the mean rate of payload mass transfer from LEO to LMO. Whereas simplified assumptions are used to allow such a quasi-analytic solution to the problem, the analysis can be used to provide a starting point quickly for more detailed analysis of specific launch opportunities.

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*Professor, Department of Aerospace Engineering.
†Graduate Student, Department of Aerospace Engineering.

Solar Sail Sizing

The fundamental measure of performance of a solar sail is its characteristic acceleration, defined as the solar radiation pressure acceleration experienced by the solar sail while oriented normal to the sun line at a heliocentric distance of 1 astronomical unit (AU). The characteristic acceleration is a function of both the efficiency of the solar sail design and the mass of the payload. At a distance of 1 AU, the magnitude of the solar radiation pressure P exerted on a perfectly absorbing surface is $4.56 \times 10^{-6}\ \mathrm{N \cdot m^{-2}}$. Therefore, allowing for the finite efficiency η of a reflecting solar sail, the characteristic acceleration a_0 is defined by[4]

$$a_0 = 2\eta P/\sigma, \qquad \sigma = m_T/A \tag{1}$$

where σ is the solar sail loading, with m_T the total mass of the solar sail and A the sail area. The sail efficiency η is a function of both the optical properties of the sail film and the sail shape due to billowing and wrinkling, with a typical value $\eta = 0.85$ (Ref. 4).

The total mass of the solar sail will now be partitioned into two components, the sail film and structure mass m_S and the payload mass m_P. Therefore, the characteristic acceleration of the solar sail may now be written as

$$a_0 = \frac{2\eta P}{\sigma_S + (m_P/A)}, \qquad \sigma_S = \frac{m_S}{A} \tag{2}$$

where σ_S is the mass per unit area of the sail assembly. This so-called sail assembly loading is a key technology parameter and is a measure of the thickness of the sail film and the efficiency of the solar sail structural design. A small, near-term demonstration mission is likely to have a sail assembly loading of order $30\ \mathrm{g \cdot m^{-2}}$, whereas initial planetary missions will require a smaller sail assembly loading of order $5–10\ \mathrm{g \cdot m^{-2}}$. However, development work to fabricate ultrathin sail films could lead to a sail assembly loading of order $1\ \mathrm{g \cdot m^{-2}}$ in the longer term.[4]

Now that the key solar sail design parameters have been defined, the process of sizing a solar sail will be considered. From Eq. (2), it can be seen that the solar sail payload mass may be written as

$$m_P = [2\eta P/a_0 - \sigma_S]A \tag{3}$$

Similarly, from Eq. (1), the total mass of the solar sail may be written as

$$m_T = 2\eta P A/a_0 \tag{4}$$

The payload mass fraction κ of the solar sail can now be defined as m_p/m_T and, thus, obtained from Eqs. (3) and (4) as

$$\kappa = 1 - a_0/\tilde{a}_0, \qquad \tilde{a}_0 = 2\eta P/\sigma_S \tag{5}$$

where $\tilde{a}_0$ is the solar sail characteristic acceleration that would be obtained with no payload. The payload mass fraction is clearly a key parameter and is a measure of the efficiency of use of the solar sail. It is clear that advances in sail technologies to reduce the sail assembly loading can be used to two ways. Such improvements can increase the solar sail characteristic acceleration, and these by reduce trip times, or, perhaps more importantly, for a fixed characteristic acceleration, can significantly increase the payload mass fraction of the solar sail.

Payload Mass Fraction Optimization

Now that the solar sail payload mass fraction has been defined, the cargo mission optimization problem can be posed. As noted earlier, the solar sail will begin its mission in LEO, from where it will spiral to Earth escape and follow a heliocentric minimum-time spiral trajectory to Mars with characteristic acceleration a_0. On arrival at Mars, the solar sail will be captured and spiral inward to LMO. The payload is then released, and the sail begins the return trip to LEO, now with a higher characteristic acceleration $\tilde{a}_0$. The performance index F to be maximized will be defined as m_p/T, where T is the total duration of the round-trip LEO–LMO–LEO trajectory, which is a function of the solar sail characteristic acceleration. By the use of Eq. (5), this performance index may be written as

$$F(a_0) = [1/T(a_0)][1 - a_0/\tilde{a}_0]m_T \tag{6}$$

By the use of Eq. (6), the extremum of the performance index may be sought by calculating the first derivative of F as

$$\frac{\partial F}{\partial a_0} = \frac{-T'}{T^2}\left[1 - \frac{a_0}{\tilde{a}_0}\right]m_T - \frac{1}{T}\frac{m_T}{\tilde{a}_0}, \qquad T' = \frac{\partial T}{\partial a_0} \tag{7}$$

where again T is a strong function of a_0. If a turning point is sought from Eq. (7), then $\partial F/\partial a_0 = 0$, so that the required characteristic acceleration and, thus, optimum payload mass fraction can be determined as

$$a_0 = \tilde{a}_0 + T/T' \Rightarrow \kappa = -(T/T')(1/\tilde{a}_0) \tag{8}$$

where the identity $\kappa = 1 - a_0/\tilde{a}_0$. has been used. It can be confirmed that the turning point is, in fact, a maximum of the performance index.

The total LEO–LMO–LEO trip time is composed of two segments with differing solar sail characteristic accelerations. For the outbound LEO–LMO trajectory, the solar sail has a characteristic acceleration a_0, whereas for the inbound LMO–LEO trajectory, the solar sail has an increased characteristic acceleration $\tilde{a}_0$ because the payload has been left in LMO. The LEO–LMO or LMO–LEO trip times can be approximated by a power law fit to minimum-time trajectories generated numerically so that the outbound trip time may be written as $T_{\mathrm{Out}} = \tilde{T}a_0^{-m}$, whereas the inbound trip time can be written as $T_{\mathrm{In}} = \tilde{T}\tilde{a}_0^{-m}$ and the total trip time $T = \tilde{T}(a_0^{-m} + \tilde{a}_0^{-m})$. The free parameters $\tilde{T}$ and m will now be determined from a fit to trajectory data generated numerically.

Minimum-time heliocentric trajectories have been generated numerically using a direct gradient optimization algorithm for transfers between the orbit of the Earth and Mars, assuming circular, coplanar orbits. A direct parameter optimization scheme was implemented with the sail steering angles specified at discrete nodes at the boundaries of segments of equal temporal displacement between zero and the terminal time. The controls were characterized across each time segment by linear interpolation between the nodes. This can then be used to transcribe the optimization problem to a nonlinear programming problem. This problem was then solved using NPSOL 5.0, a FORTRAN77 package based on sequential quadratic programming.[5]

It can be shown numerically that the outbound and return trajectories have almost exactly the same duration for a given characteristic acceleration with the assumptions used. The planet centerd escape and capture times have been determined using a sail steering law that maximizes the instantaneous rate of change of orbit energy. Occultation of the sail has not been included because this is launch-date specific, but inclusion of occultation will only lengthen escape and capture spirals by ~2–3%. Again, the escape and capture times are the same for a given body and characteristic acceleration. The starting orbit in LEO is taken as a circular orbit at an altitude of 1000 km (to avoid air drag), whereas the final orbit in LMO is taken as a circular orbit at an altitude of 500 km. By the use of these data, the parametric relationship between trip time (days) and solar sail characteristic acceleration ($\mathrm{mm \cdot s^{-2}}$) yields $\tilde{T} = 802$ days and $m = 0.647$, as shown in Fig. 1. Again, particular launch opportunities are not being considered so that the trajectory data are representative of actual trip times.

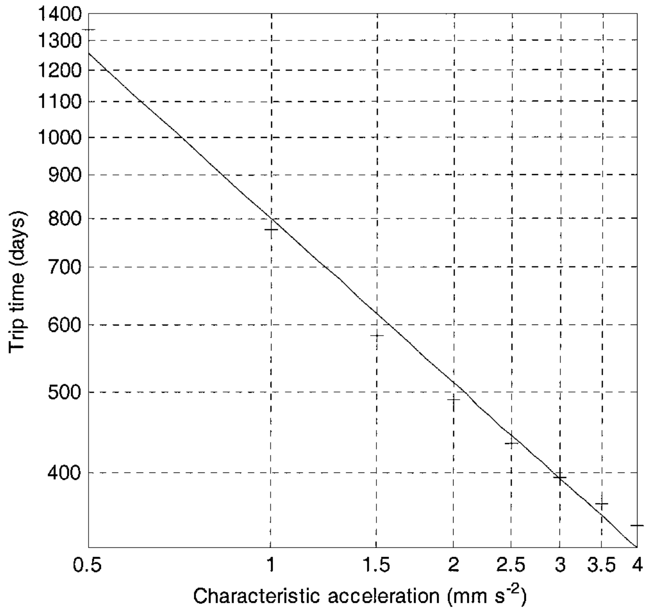

Fig. 1 LEO–LMO (and LMO–LEO) duration as a function of solar sail characteristic acceleration: +, trajectory data points and ——, least squares fit.

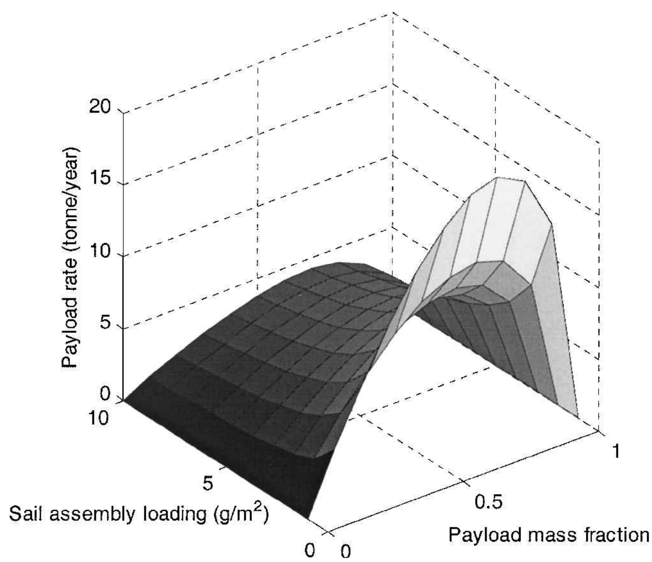

Fig. 2 Mean payload mass transfer rate for a total solar sail mass (sail and payload) of 50 t.

Substituting the power law into Eq. (8) yields an implicit relationship from which the optimum payload mass fraction can now be determined:

$$(1 - \kappa)(1 + 1/m) + (1/m)(1 - \kappa)^{1+m} - 1 = 0 \tag{9}$$

For $m = 0.647$, Eq. (9) then yields the optimum payload mass fraction as 0.694. The solar sail should, therefore, be loaded at approximately 70% mass fraction to ensure that the mean rate of payload mass transfer is maximized. The relationship between payload delivery, payload mass fraction, and sail assembly loading (sail technology level) is shown in Fig. 2 for a single solar sail with a total mass of 50 t. It can be seen that the optimum payload mass fraction of 0.694 is evident and that the payload mass delivery rate is a strong function of the sail assembly loading. The sail technology level will, therefore, have a major effect on payload mass delivery rate, whereas the mass delivery rate is somewhat less sensitive to the payload mass fraction, again, as seen in Fig. 2.

Sail sizing for a 50-t solar sail is also given in Table 1 for a range of sail assembly loadings. Each solar sail has an optimum payload mass fraction of 0.694 and, thus, delivers a payload of 34.7 t. It can

Table 1 Sizing for a total solar sail mass (sail and payload) of 50 t and sail efficiency $\eta = 0.85$

	σ_s, $g \cdot m^{-2}$		
Parameter	2	4	6
$\tilde{a}_0$, $mm \cdot s^{-2}$	3.88	1.94	1.29
a_0, $mm \cdot s^{-2}$	1.19	0.59	0.40
Round-trip time, days	1052	1647	2141
Sail side, km	2.77	1.96	1.60

be seen that extremely large sails are required for cargo missions that deliver payloads of this size. In addition, a low sail assembly loading is required to deliver payload mass at a reasonable rate. For a high-performance solar sail with a sail assembly loading of 2 $g \cdot m^{-2}$, a 34.7-t payload is delivered every 2.9 years, corresponding to a mean delivery rate of 12 t per year. Clearly, a fleet of several solar sails would be required to deliver large payloads at a greater than annual rate. The phasing of such a fleet of solar sails will depend on particular launch opportunities, which are not considered here.

Conclusions

A simple solar sail mass model has been used with parametric trajectory data to determine the optimum solar sail payload mass fraction for a Mars cargo mission. It has been found that the solar sail should be loaded with a payload mass fraction of approximately 70% to ensure that the mean rate of payload mass transfer is maximized. Although the analysis has assumed simplified trajectories, and, thus, ignored the vagaries of particular mission opportunities, it does indicate an optimum solar sail payload mass fraction that will maximize the mean payload mass delivery rate to Mars. Finally, a similar analysis can also be performed for solar sails utilized for orbit transfer vehicles delivering payload to geosynchronous orbit, for example.

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J. A. Martin
Associate Editor